Carrier Tracking by Smoothing Filter Can Improve Symbol SNR

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The potential benefit of using a smoothing filter to estimate carrier phase over use of phase locked loops (PLL) is determined. Numerical results are presented for the performance of three possible configurations of the DSN Advanced Receiver. These are Residual Carrier PLL, Sideband Aided Residual Carrier PLL, and finally Sideband Aiding with a Kalman Smoother. The average symbol SNR after losses due to carrier phase estimation error is computed for different total power SNRs, symbol rates and symbol SNRs. It is found that smoothing is most beneficial for low symbol SNRs and low symbol rates. Smoothing gains up to 0.4 dB over a Sideband Aided Residual Carrier PLL, and the combined benefit of Smoothing and Sideband Aiding relative to a Residual Carrier Loop is often in excess of 1 dB.

I. Introduction

Smoothing as a way to improve carrier tracking has been proposed and analyzed (Ref. 1). It has been shown that by cascading an optimum Kalman smoother with a carrier tracking loop, up to a 6-dB reduction in the phase error variance can be obtained with a second order smoother. This work presents the potential improvement in the effective data signal-to-noise ratio (SNR) by using a smoother in the DSN Advanced Receiver. Optimum modulation indexes and sideband aiding are used.

The average symbol SNR, after degradation due to carrier phase estimation error (radio loss), is used as a measure of performance when comparing the various possible configurations for the receiver. Also, conditions such as the minimum SNR needed to ensure acquisition and lock of the loops are satisfied in each case.

II. Analysis

The analysis is carried out for three possible configurations of the DSN Advanced Receiver. They are:

(1) Residual Carrier tracking only (RC)

(2) Residual Carrier plus Sideband Aiding (SA)

(3) SA plus Smoothing (SM)

The received signal for the DSN Advanced Receiver is assumed to be of the form (Ref. 2)

$$r(t) = \sqrt{2P}\ \sin(\omega_i t + \Delta\, D(t) + \Theta_c) + n(t) \qquad (1)$$

where

$r(t)$ = received signal (v)

P = average signal power (v^2)

Δ = modulation index (rad)

$D(t)$ = $d(t)\ \mathrm{sgn}\ (\sin(\omega_{sc} t + \Theta_{sc}))$

$d(t)$ = $\sum_{\ell - \infty}^{\infty} a_\ell p(t - \ell T)$, $a_\ell = \pm 1$ with equal probability

$p(t)$ = 1 for $0 < t \leqslant T$, 0 elsewhere

ω_i = received IF frequency (rad/s)

Θ_c = carrier phase (rad)

ω_{sc} = subcarrier frequency (rad/s)

Θ_{sc} = subcarrier phase (rad)

$n(t)$ = $\sqrt{2}\, n_c(t) \cos(\omega_i t + \Theta_c) - \sqrt{2}\, n_s(t) \sin(\omega_i t + \Theta_c)$ is a noise process with $n_c(t)$ and $n_s(t)$ being statistically independent, stationary, band-limited white Gaussian noise processes with one-sided spectral density N_0 (v^2/Hz) and one sided bandwidth W.

T = symbol time (s)

The carrier phase estimate is $\hat{\Theta}_c$, and the resulting carrier phase error is $\phi = \hat{\Theta}_c - \Theta_c$. It is assumed that the phase error process $\phi(t)$ is essentially constant during one symbol time. Thus, for example, in the RC case the demodulated waveform is

$$z(t) = \sqrt{P}\ \sin \Delta D(t) \cos \phi(t) + n'(t) \qquad (2)$$

where $n'(t)$ is a narrow-band Gaussian process with one-sided spectral density N_0.

Conditioned on $\phi(t)$, the symbol SNR at the output of the symbol integration is

$$\frac{E_s(\phi)}{N_0} = \frac{PT \sin^2 \Delta \cos^2 \phi}{N_0} \qquad (3)$$

and the average symbol SNR after losses due to ϕ, $[E_s/N_0]_{\text{ave}}$, can be approximated by

$$\left[\frac{E_s}{N_0}\right]_{\text{ave}} = \frac{PT \sin^2 \Delta}{N_0} (1 - \sigma_\phi^2) \qquad (4)$$

where σ_ϕ^2 is the phase error variance. In Eq. (4) it is assumed that σ_ϕ is small ($\ll$ 1 rad). This is valid for all the cases discussed here; thus, for residual carrier tracking only, from Ref. 2

$$\sigma_{\phi_1}^2 = \frac{N_0 B_L}{P \cos^2 \Delta} \qquad (5)$$

where B_L is the single-sided loop bandwidth.

For residual carrier tracking the modulation index which maximizes the average symbol SNR is independent of the symbol SNR and satisfies $\cos^4 \Delta = N_0\, B_L/P$, under the above conditions. This follows from substituting Eq. (5) into Eq. (4) and maximizing.

For sideband aiding,

$$\sigma_{\phi_2}^2 = \frac{1}{\dfrac{P}{N_0\, B_L}\left[\cos^2 \Delta + \dfrac{\sin^2 \Delta}{1 + \dfrac{1}{(2E_s/N_0)}}\right]} \qquad (6)$$

where

$$E_s = PT \sin^2 \Delta \qquad (7)$$

Finally, for a smoothing estimator following a loop with RC plus SA, the best possible result with a second order smoother is (Ref. 1)

$$\sigma_{\phi_3}^2 = \sigma_{\phi_2}^2 / 4 \qquad (8)$$

The average symbol SNR depends on the modulation index Δ. Therefore, it is necessary to find the best Δ for each case. This best Δ is that which maximizes $[E_s/N_0]_{\text{ave}}$ subject to the constraint of sufficient energy in the unsmoothed carrier loop to ensure acquisition and lock, without "too many" cycle slips.

One possible way to restrict the modulation index would be to assume that there is sufficient carrier power to lock on the residual carrier only loop. This restriction leads to an easy acquisition procedure, but not to the best effective SNR. We therefore use a criterion for RC plus SA locking, which is described as follows. The maximum rms phase error which can be tolerated without too many cycle slips is related to the average phase detector output versus ϕ, which is called the S-curve. For sideband aiding, the S-curve can be approximated (Ref. 2)

$$S(\phi) = \sqrt{2P}\left[\cos\Delta \sin\phi + \frac{\sin\Delta}{4} \sin^2\phi\right] \tag{9}$$

The value of ϕ for which $S(\phi)$ is maximum can be found to be

$$\phi_{max} = \cos^{-1}\left[\frac{-\cos\Delta + \sqrt{1+\sin^2\Delta}}{2\sin\Delta}\right] \tag{10}$$

In a conventional PLL, acceptable cycle slipping and acquisition is usually achieved with $\sigma_\phi^2 \leqslant 0.2$ (loop SNR $\geqslant 7$ dB). Since the corresponding S-curve peaks at $\phi_{max} = \pi/2$, this roughly amounts to

$$\sigma_\phi \leqslant 0.285\, \phi_{max} \tag{11}$$

We use as this criterion for sideband aided loops with ϕ_{max} according to Eq. (10). A numerical analysis is then carried out to maximize $[E_s/N_0]_{ave}$ from Eqs. (4) and (6) subject to the constraint of Eq. (11).

III. Results

The maximum (over Δ) average symbol SNR, $[E_s/N_0]_{ave}$, was evaluated for the various tracking methods as a function of the ratio of total signal power to noise in the loop bandwidth, P/N_0B_L. The results are shown in Fig. 1 for E_s/N_0 of 0 dB, -3 dB, -10 dB, and -20 dB, respectively. Also shown on the curves are the modulation indexes which maximize the average SNR. Note that, using Eq. (7), the symbol rate for any point on the curves can be determined by:

$$R_s = \frac{1}{T} = \frac{P}{N_0B_L} \; \frac{B_L \sin^2\Delta}{E_s/N_0} \tag{12}$$

Thus, symbol rate, R_s, is almost proportional to the abscissa value in Fig. 1.

For the higher symbol SNRs, Figs. 1(a) and 1(b), the use of sideband aiding with optimized modulation index results in almost 100 percent efficient utilization of received power, except at very low symbol rates. There is little to gain by use of smoothing, except at low symbol rates. At E_s/N_0 = -20 dB, Fig. 1(d), sideband aiding does not improve performance very much, but smoothing helps significantly. Both SA and SM are significant at -10 dB, Fig. 1(c).

The benefits of sideband aiding relative to residual carrier tracking only are shown versus P/N_0B_L, in Fig. 2, for the four different symbol SNRs. For the higher SNRs, 0 dB to -3 dB, the benefit is often greater than 1 dB and sometimes as much as 2.5 dB, at low P/N_0B_L. The benefit is almost 0.4 dB at the highest P/N_0B_L shown, 500, which corresponds to a symbols rate of 500 B_L and 1000 B_L at symbol SNRs of 0 dB and -3 dB, respectively. At a symbol SNR of -10 dB, the maximum benefit of sideband aiding is approximately 0.65 dB, and occurs approximately at P/N_0B_L = 100. At a symbol SNR of -20 dB, sideband aiding gains less than 0.1 dB for all P/N_0B_L shown.

The benefits of smoothing over sideband aiding are shown in Fig. 3. At the lowest P/N_0B_L shown, 20, the benefits are 0.38 dB, 0.37 dB, 0.29 dB and 0.25 dB at symbol SNRs of -20 dB, -10 dB, -3 dB and 0 dB, respectively. The corresponding symbol rates are 1100 B_L, 124 B_L, 33 B_L and 18 B_L. Looking at the results from another viewpoint, smoothing gains at least 0.25 dB over sideband aiding at all symbol SNRs, provided that the symbol rate is low enough, i.e., less than 18 B_L, 52 B_L, 920 B_L and 1400 B_L for symbol SNRs of 0 dB, -3 dB, -10 dB and -20 dB.

IV. Discussion of Other Conditions

Results have been presented only for cases in which the modulation index is optimized, subject to a lock constraint, and in which sideband aiding is used. For any conditions, smoothing can reduce the carrier phase error variance, and hence the radio loss, by a factor of four. Thus, smoothing is a valuable tool whenever the radio loss is large. For example, suppose that sideband aiding is not used and the modulation index and loop bandwidth are such that the carrier loop SNR is 7 dB. Then the radio loss is approximately 0.97 dB without smoothing, and 0.22 dB with smoothing, a gain of 0.75 dB. On the other hand, if the modulation index for a spacecraft is always low enough so that the residual carrier loop SNR is high and the radio loss is low, then there is minimal potential benefit to either sideband aiding or smoothing.

Smoothing is also beneficial in reducing the effects of oscillator noise and spacecraft dynamics, as shown in Ref. 1. This might lead to reduction in the loop bandwidth, and thus further reduction in radio loss. The impacts on symbol SNR have not yet been evaluated.

V. Implementation, Cost and Remaining Problem

A block diagram for carrier smoothing is given in Ref. 1. It is estimated that this can be implemented with one special purpose digital board, plus one single board computer, added to the Advanced Receiver. The cost is estimated at \$5K to \$10K per receiver.

The main problem remaining is the possible effect of nonlinearities in the phase detectors on the smoothing filter phase solution. This needs to be evaluated by simulation before a breadboard is developed.

VI. Conclusions

Smoothing is a useful tool for carrier tracking at low symbol SNRs and low data rates. For optimum modulation indexes and using sideband aiding, smoothing virtually eliminates radio loss and gains up to 0.4 dB in average effective symbol SNR. Larger gains in SNR are possible in cases in which sideband aiding is not used, or in which the modulation index is not optimized.

References

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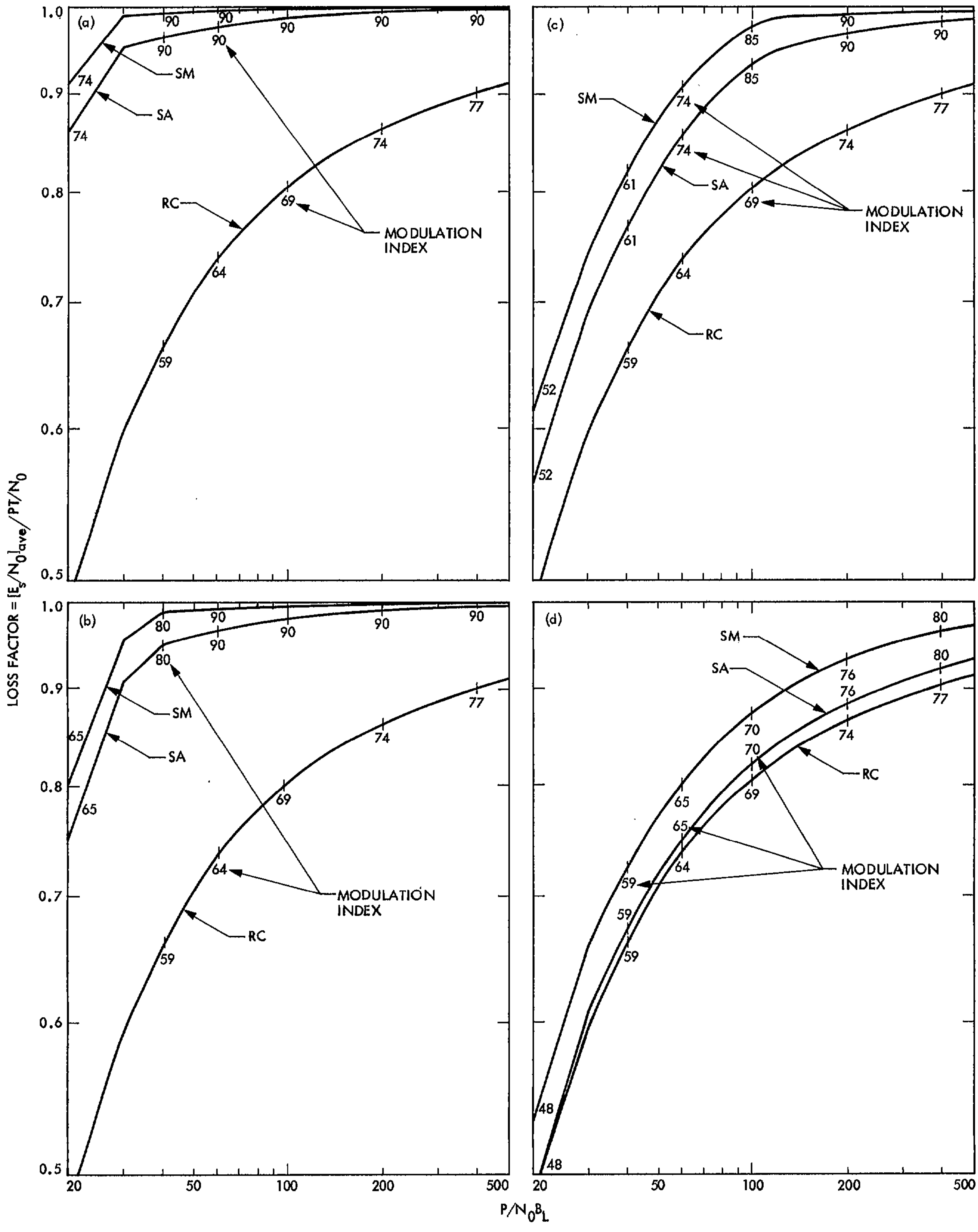

Fig. 1. Comparison of carrier phase tracking loss factor vs total power SNR in loop bandwidth. The symbol SNR is (a) 0 dB, (b) −3 dB, (c) −10 dB, and (d) −20 dB.

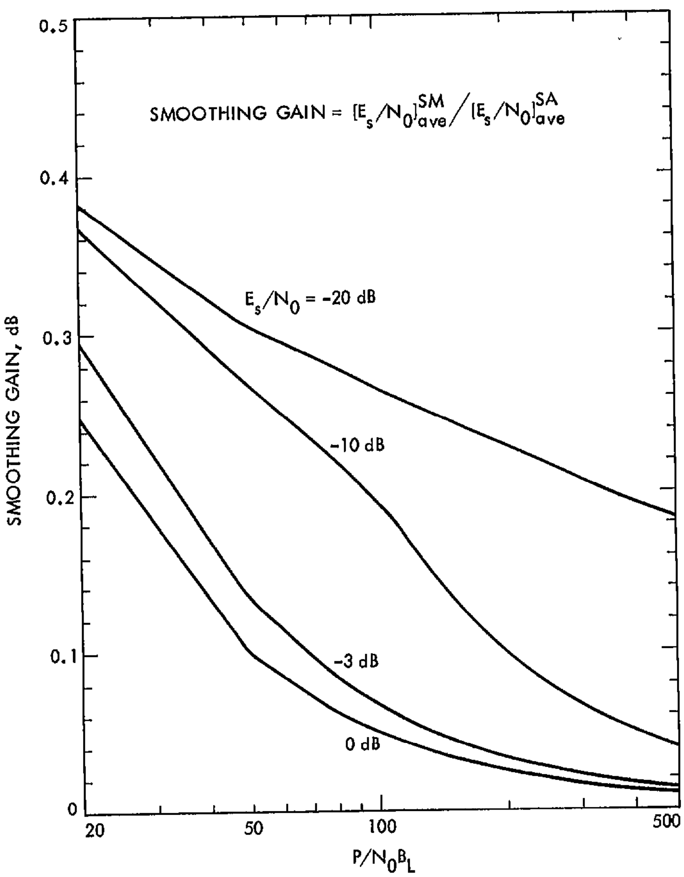

Fig. 2. The gain of sideband aiding over residual carrier tracking

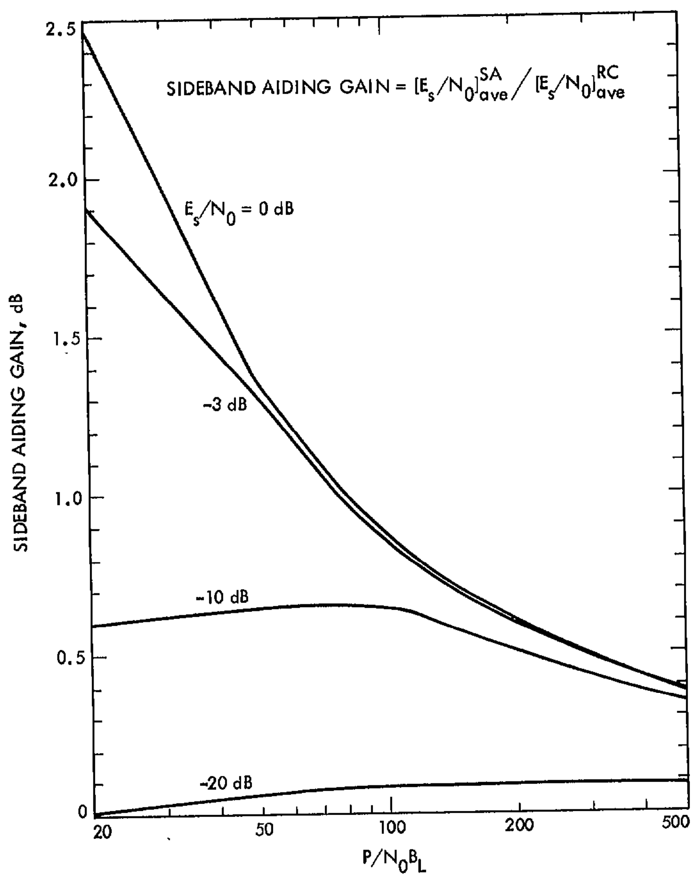

Fig. 3. The gain of smoothing over sideband aiding